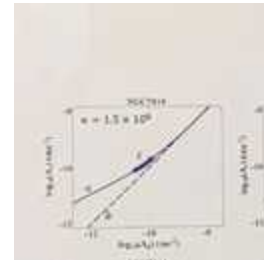


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Study of Dynamic Structure of Galaxies

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Tsutomu Kambe¹, Masanori Hashiguchi²

¹University of Tokyo (Former Professor); ²Guest Scholar at MIMS, Meiji University, Tokyo, Japan.

Tsutomu Kambe: Guest Scholar at MIMS, Meiji University, Tokyo, Japan. MIMS=Meiji Institute for Advanced Study of Mathematical Science e-mail: kambe@ruby.dti.ne.jp;



Tsutomu Kambe

University of Tokyo (Former Professor)

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Unraveling of Dynamic Structure of Galaxies and Dark Matter Effect, based on General Relativity

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Abstract

This is a novel approach based on general relativity theory to the galactic dark-matter effect observed in rotating galaxies,. Dynamical unfolding within galaxies has been investigated to find a possible physical mechanism producing the flat rotation curve observed in outer parts of galaxies. Dynamical structures are studied for typical galaxies, based on fluid dynamics extended to the relativistic theory. The results show an excellent match of the present predictions with the observed finding by McGaugh, Lelli & Schombert (2016). This matching implies that the present approach has captured an essential aspect of the galactic dark-matter effect. Concerning the physical mechanism, this study implies that the orbital hyperspeed of order $40 \sim 200$ km/sec observed in the halos is maintained by a new force of fluid Lorentz force working in galactic space. The existence of fluid Lorentz force is not contradicting with the concept of the theory of general relativity. Spiral galaxies, Dark matter effect, Dynamic structure, General relativity

Attachments



Descriptions of the ...

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Present study investigates physical mechanisms and dynamical interplays of the galaxies, on the basis of the general theory of relativity and Fluid Mechanics. *Before starting our study, the following two well-known facts are particularly noted:*

1 Gas clouds exist abundantly in every galaxy like an atmosphere

Typical galaxies are surrounded with gas-rich atmospheres. Those galaxies show the Dark Matter Effect. Dynamic structures of the galaxies are the target of the recent studies of [3, 4], based on General Relativity. Observational facts known before the start of the present study are as follows [1, 2]:

(i) There exists abundant neutral hydrogen gas in the dark halo. Neutral hydrogen atoms are observed

by the HI-line of wavelength 21-cm (hence optically dark and unseen photographically).

(ii) Observed rotation curves of galaxies are nearly flat (i.e. constant) well outside luminous regions.

(iii) Unseen objects (i.e. neutral gas clouds) are moving at very high speeds of the order 40 ~ 200

km/sec in the halo of galactic space.

[1] Hoekstra, H., van Albada, T. S. & Sancisi, R. On the apparent coupling of neutral hydrogen and dark

matter in spiral galaxies. *Mon. Not. R. Astron. Soc.*, 323, 453-459, (2001).

[2] Bosma, A. 21-cm line studies of spiral galaxies. II. The distribution and kinematics of neutral hydrogen

in spiral galaxies of various morphological types. *Astron. J.*, 86, 1825-1846, (1981).

[3] Kambe, T. Relativistic Exploration of Dark Matter Effects in Rotating Galaxy, Studied Fluid Dynamically.

Glob. J. Sci. Front. Res. A Physics & Space Science, 23 (8), 13 - 28 (2023).

[4] Kambe, T. & Hashiguchi, M. Unlocking. Galactic Mysteries: Relativistic Insight into Orbital Hyper-peeds

and Dark Matter in Gas-Rich Galactic-Halos. ***Glob. J. Sci. Front. Res. A Physics & Space Science***, 23

(10), 1 - 12 (2023).

2 Rotational motions of spiral galaxies are in general non-Keplerian

The observed finding of McGaugh, Lelli & Schombert [5] states that the rotation curves of most galaxies are non-Keplerian. Namely those are different from the rotation curve predicted by the Kepler's law for Solar planets. They have clarified the non-Keplerian rotation curves of galaxies from statistical analysis of 163 galaxies.

[5] McGaugh, S., Lelli, F. & Schombert, J. Radial Acceleration Relation in Rotationally Supported

Galaxies. **Phys. Rev. Lett.** 117, 201101 (2016).

In the paper [5], no particular model is proposed whether non-Keplerian nature is due to the dark matter or the result of new dynamical laws. The recent studies [3, 4] interpreted the galactic dark matter issue without assuming unknown dark matters by a physical mechanism caused with high speed motions of gas clouds in the galactic atmosphere.

The present theory to be given below from §3 onward is an attempt to clarify the observed non-Keplerian nature described in §2 with a new physical effect caused by the abundant galactic atmosphere moving at very high speeds described in §1.

Main part: Scenario of the present study and Theoretical formulation.

3 Formulation of the Present Study:

Formulation of the present study is based on the General Relativity and the Fluid Mechanics. A theoretical approach is taken to investigate the *dynamical structure* of Galaxies by combining two fields of physics: Theory of general relativity and Fluid mechanics. The latter is strengthened by the *Fluid Gauge Theory* [13] (see §3.5 below), formulated relativistically by taking account of a new physical effect generated by gas clouds moving at very high speeds, in its broadest meaning of relativity.

[13] Kambe, T. Fluid Gauge Theory, **Glob. J. Sci. Front. Res** -A, 21 (4), 113 – 147, (2021).

3.1 **Dynamical mechanism is played interactively by two kinds of forces in galaxies**

One is the gravity force, needless to say, and the other is a fluid Lorentz force that is new. The two forces switch their roles effectively to keep the orbital velocity nearly constant in the galactic space [3, 4].

3.2 **Existence of the fluid Lorentz force is not contradicting with the concept of the theory of relativity.**

Existence of the fluid Lorentz force is not contradictory with the concept of the theory of general relativity. In fact, according to Einstein (1914) [7], the gravity field has the character of a tensor field, and the theory of general relativity makes it probable that mutual action of matters existing in the space is an important player of galaxy dynamics. This complies with the principle of relativity in its broadest meaning. The mutual action enables the interplay between the two different dynamical effects in the rotation dynamics in galaxies. This gives rise to the dark matter effect of unseen objects which are moving at hyper-speeds of the order 40 ~ 200 km/sec in the halo of galactic space.

[7] Einstein, A. The formal foundation of the general theory of relativity," **Sitzungsber. Preuss. Akad.**

Wiss. Berlin (Math.Phys.), 1030 – 1085, (1914).



3.3 **The present study is based on the Hilbert's variational formulation**

Present relativistic study is based on the theory of Hilbert (1915) [9]. Its variational formulation gives the same field equation of Einstein (1914) [7]. Einstein developed his theory so as to comply with the principle of relativity in the broadest meaning of the word, epistemologically according to [7]. The system of equations is general and covariant under arbitrary transformations of the coordinates.

On the other hand, Hilbert's theory [9] has a deductive structure, but also his theory led to the same field equation as Einstein presented. It was just after the Einstein's paper [8]. Now that, the Hilbert theory has become an essential constituent of the theory of General Relativity [10].

[8] Einstein, A. Zur allgemeinen Relativitätstheorie. **Sitzungsber. Preuss. Akad. Wiss. Berlin**, 778 – 786, (1915).

[9] Hilbert, D. Die Grundlagen der Physik (Erste Mitteilung). **Nachr. Gesell. Wiss. Göttingen, Math.-Phys.**, 395 - 408 (1915).

[10] Renn J. & Stachel, J. Hilbert's Foundation of Physics: From a Theory of Everything to a Constituent of General Relativity. **Physics, Boston studies in the philosophy of Science**, 1 - 41 (2007).

3.4 **Importance of the Hilbert's formulation :**

The reasons why the present formulation adopts the Hilbert theory are as follows. One of its advantages lies in the deductive and variational formulation. Another merit is the fact that the electro-magnetic field is incorporated in the Lagrangian formulation and coupled with the gravitational field. Hilbert specified only general property of the electro-magnetic Lagrangian, requiring it be a generally invariant scalar under Lie derivatives with respect to arbitrary vector field.

The present study proposes the Fluid Gauge Theory [13] to be incorporated in the theoretical frame of general relativity and coupled with the gravitational field. In other words, the Hilbert's approach can be generalized to neutral fluids as well.

3.5 **, Fluid Gauge Theory (FGT)**

[13] T.Kambe, Fluid Gauge Theory, *Glob. J. Sci. Front. Res -A*,⁺ 21 (4), 113, (2021).

According to Kambe (2021) [13], there is a transition of stress field within the flow field when the flow field is intensified and becomes rotational in the fluid flows. The stress field is **isotropic** before the transition. It becomes **anisotropic** after the transition. One can introduce a 3-vector ***a*** for the fluid gauge field after the transition. The transition is analogous to the *spontaneous symmetry breaking* known in the field theory [14, 15].

Within the framework of the special relativity, one can write an FGT-equation of motion for the fluid motion [3]. Taking account of the FGT-term of the *fluid Lorentz force* together with *gauge-field* 3-vectors **b** (fluid magnetic field) and **e** (fluid electric field), the governing equations are given as follows:

$$\rho [\nabla_t \mathbf{v}] = [\text{Stress force}] + [\text{Fluid Lorentz force}] \quad \nabla_t \mathbf{v} : \text{covariant derivative w.r.t. } t$$

where t is the time, $\mathbf{v}$ the fluid velocity, ρ the mass density, $[\text{Fluid Lorentz force}] = \rho (\mathbf{v} \times \mathbf{b})$,

and the anisotropic stress is a fluid version of the electromagnetic Maxwell stress.

[14] Djouadi, A. The anatomy of electro-weak symmetry breaking, I: The Higgs boson in the standard

model. **Phys. Rept.** 457 1-216 (2008).

[15] Nambu, Y. Spontaneous Symmetry Breaking in Particle Physics: A Case of Cross-Fertilization,

Nobel Lecture (2008).

4 **Present study: Action principle, Lagrangian and Balance equation deduced**

The Lagrangian of the present variational formulation consists of two parts:

- (i) Lagrangian L_g describing a curved space (by gravity) and built of geometry alone;
- (ii) Lagrangian L_f describing a contribution from dynamical flows of gas clouds existing in galactic space

like a gaseous atmosphere and treating it as a continuous fluid.

The action principle leads to the Einstein's field equation:

$$G_E = k T_S : \quad k = 8\pi G/c^4, \quad G: \text{gravity constant, } c: \text{light}$$

velocity,

where G_E and T_S are (4, 4)-tensors with G_E the Einstein curvature tensor and T_S the stress energy tensor of a fluid motion. The tensor T_S is strengthened by a Fluid Gauge Theory with a combined stress tensor including a fluid version of the Maxwell stress.

4.1 **In weak field limit under steady state,**

In a weak field limit, the dynamical motion of the galactic fluid is described by a balance equation of acceleration. It is remarkable that the mass density is dropped out in the equation of motion apparently (see the equation (6) of the main article). Hence, the equation is equivalent to an equation of free motion of a point mass.

The mass density dropped out apparently in the equation, although the gravitational potential and the fluid magnetic field **b** depend on the mass density implicitly (see the equations (13) and (15) of the main article). Defining the fluid gauge field 3-vector by **a**, the fluid magnetic field is defined by $\mathbf{b} = \nabla \times \mathbf{a}$.

In order to capture the significance of this fact more deeply and gain useful results, the problem is simplified without losing essential physics. Consider a circular disk-galaxy rotating around the center *in steady-state*, and suppose that the galaxy keeps its axisymmetric form, represented with cylindrical polar coordinates (Z, R, φ), where the plane $Z=0$ coincides with the disk plane, described with the polar

coordinates (R, φ) , the galaxy center being at $R=0$.

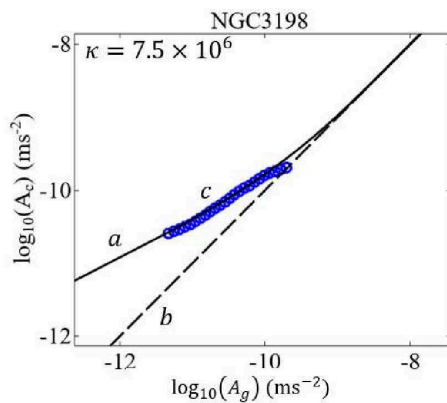
Under the simplified situation, we have $\mathbf{v} = (0, 0, V)$ and $\mathbf{a} = (0, 0, A)$, both having only azimuthal

φ -component. The present formulation reduces to an equation of acceleration balance,

$$A_c - A_g = A_f,$$

among three components of acceleration, where $A_c = V^2/R$ (V : velocity, R : radial distance) is the centripetal acceleration, A_g the acceleration of the gravitational force, and $\mathbf{A}_f$ the acceleration generated by **fluid Lorentz force** given by $-(\mathbf{v} \times \mathbf{b})_R$ (radial component).

- 4.2 If $\mathbf{A}_f = \mathbf{0}$, then we have $A_c = A_g$, denoted in the diagram given below by the diagonal line $\mathbf{b}$ forming a 45 degree angle with the abscissa. This denotes the Keplerian motion. In most galaxies, observations show that A_f is not zero according to McGaugh-Lelli-Schombert (2016) [5]. The curve $\mathbf{a}$ was found by their study.



the ordinate ($\log_{10}(A_c) \text{ ms}^{-2}$) and the abscissa ($\log_{10}(A_g) \text{ ms}^{-2}$)

- 4.3 The present theoretical study has deduced an amazing result (shown by the blue sequence of dots marked with $\mathbf{c}$ in the above diagram) that agrees quite encouragingly with the curve $\mathbf{a}$ in the above diagram found by McGaugh, Lelli & Schombert (2016) [5] from the astronomical observations of more than a hundred of galaxies. **The agreement is in fact remarkable.**
- 5 **Present analytical approach has been reduced to the three equations (9), (13) and (15) of the main article.**



In the present study, the observed rotation curve $V_{\text{obs}}(R)$ plays an essential role to arrive at the final results. The velocity data are acquired accurately in a comparative sense because the data are directly obtained from the spectral analysis, whereas the mass density data are estimated indirectly through the mass-to-light ratio

It is noteworthy to find that the present simplified model of galaxy dynamics is reduced to the three equations, given by the equations (9), (13) and (15) of the main article.

(I) Eq.(9) is equivalent to $A_c - A_g = A_f$, considered in §4.1, which can be rewritten as

$$V^2/R - \partial_R \Phi_g = (v \times b)_R, \quad \text{where} \quad (v \times b)_R = (V/R) \partial_R (RA),$$

$$\Psi = RA.$$

$$(II) \text{ Eq.(13): } R^{-1} \partial_R (R \partial_R \Phi_g) = 4\pi G \rho_0(R),$$

which is derived from the Poisson equation for the gravity potential Φ_g : $\nabla^2 \Phi_g = 4\pi G \rho$,

$$(III) \text{ Eq.(15): } \partial_R (R^{-1} \partial_R \Psi) = -\mu_* \rho_0(R) V(R),$$

which is derived under the condition $\mathbf{v} = (0, 0, V)$ and $\mathbf{a} = (0, 0, A)$, from the fluid Ampere's law: $\nabla \times \mathbf{b} = \mu_* \rho \mathbf{v}$, obtained from the equation (4) of the main article deleting the term of time derivative. Noting that only non-zero component of $\nabla \times \mathbf{b} = \mu_* \rho \mathbf{v}$ is azimuthal, because

$\nabla \times \mathbf{b} = \nabla \times (\nabla \times \mathbf{a}) = -\nabla^2 \mathbf{a}$ and its azimuthal component is given by $-\partial_R (R^{-1} \partial_R (RA))$ with $\Psi = RA$.

5.1 Finally, one obtains a single differential equation,

$$R \partial_R (R^{-1} \partial_R \Psi) = -\lambda c^{-2} V(R) \partial_R (V^2 + V \partial_R \Psi), \quad \lambda = \mu_* (4\pi G)^{-1} c^2$$

(dimensionless)

from the above three equations. In fact, among the three equations of (I) ~ (III), the gravity term $\partial_R \Phi_g$ can be eliminated between (I) and (II). Next, between the resulting equation and (III), the density $\rho_0(R)$ can be eliminated. Thus, we obtain the above single equation in which the velocity V is given by the observed rotation curve $V_{\text{obs}}(R)$.

6 Methods of Numerical Computation

The differential equation to be solved numerically is the equation given in the previous §5.1, which is converted to the following dimensionless form EQG:

$$(EQG): \quad R_* \partial_{R_*} (R_*^{-1} \partial_{R_*} \Psi_*) = -\lambda V_* \partial_{R_*} (V_*^2 + V_* \partial_{R_*} \Psi_*),$$

where R_* , V_* and Ψ_* are dimension-less variables defined by $V_* = V_{\text{obs}}/c$, $R_* = R/L$ and

$\Psi_* = \Psi/(cL)$ with L a scale parameter characterizing the galaxy under investigation. The equation (EQG) is a 2nd order ordinary differential equation with the dimension-less parameter λ .

This equation describes the mechanism of how the gauge potential Ψ_* is excited by the rotation velocity $V_*(R_*)$ of the galaxy. The degree of excitation is controlled by the dimension-less parameter

$\lambda = (\mu_*/G) (4\pi)^{-1} c^2$, proportional to the ratio μ_*/G , signifying relative strength of two excitation mechanisms represented by two parameters μ_* and G . The μ_* denotes the excitation by the current flux $\rho_0 V$ (see the equation (III) of §5) and the constant G denotes the excitation by the gravity. The equation (EQG) is solved by the **finite-element scheme**.

Regarding the boundary conditions, firstly, the radial section of R is chosen where $R \in [R_1, R_2]$ with

$R_1 < R_2$. Since the present study is interested in investigating the halo, the R_2 -value is chosen within the halo. Then we have $V_2 = V_{\text{obs}}(R_2)$ of the observed rotation velocity, and $A_c(R_2) = V_2^2 / R_2$.

Given the value $A_c(R_2)$, the equation $A_c = F(A_g)$ of the main article (2) yields $A_g(R_2) = F^{-1}[A_c(R_2)]$

by the inversion. Then, the first derivative of Ψ at R_2 is given by (16) of the main article as follows:

$$d\Psi/dR|_{R_2} = (-A_c(R_2) + A_g(R_2)) (R_2/V_2) \equiv Y_2$$

denoted as Y_2 . In the diagram of §4.2, although the point $(A_g(R_2), A_c(R_2))$ is located on the curve given by $A_c = F(A_g)$, the other points are not necessarily located on the same curve, because the other points are determined by solving the differential equation (EQG). At the other end R_1 , the condition

$\Psi(R_1) = 0$ is imposed to fix the arbitrariness of Ψ -value.

Initially, an appropriate value λ_1 is chosen for the unknown parameter λ of (EQG), and the rotation curve is given by the observed function $V_{\text{obs}}(R)$. Then, the two boundary conditions on both sides of the R -section are given by

$$(BC): \quad \Psi_* = 0 \quad \text{at} \quad R_{*1} = R_1/L; \quad \partial_{R_*} \Psi_* = Y_{*2} \quad \text{at} \quad R_{*2} = R_2/L$$

where $Y_* = Y/c$.

To solve the second order differential equation (EQG), the initial turn of the trial computation is carried out by the *finite-element scheme* for the above conditions (BC), and one finds the first trial function

$\Psi^{(1)}(R)$ for $R \in [R_1, R_2]$.

From $\Psi^{(1)}(R)$, the equation (18) of the main text gives a trial value $A_{\text{DM}}^{(1)} = |VR^{-1} \partial_R \Psi^{(1)}|$ at R .

Then from (17) of the main article, the variable A_{grav} (written in place of A_g) is given a trial value by

$A_{\text{grav}}^{(1)}(R) = V_{\text{obs}}^2(R)/R - A_{\text{DM}}^{(1)}$, where $A_{\text{grav}}^{(1)}$ is regarded as the first trial value of $|\partial_R \Phi_g|$ at the position R , together with $A_c^{(1)}(R) = A_{\text{grav}}^{(1)} + A_{\text{DM}}^{(1)}$. In general, the diagram of MLS-plot gives A_{DM} once the abscissa A_g -value is given. Setting the value of A_g by the computed value $A_{\text{grav}}^{(1)}$ at R , one must compare the computed trial value $A_{\text{DM}}^{(1)}$ with the value $A_{\text{DM}} = F(A_{\text{grav}}^{(1)}) - A_{\text{grav}}^{(1)}$. Usually, both are different. Note that $F(A_{\text{grav}}^{(1)}) \neq A_c^{(1)}(R)$.

If those two of $A_{\text{DM}}^{(1)}$ and A_{DM} do not match, the index-parameter λ_1 must be corrected to a new better value λ_2 , and another try of computation must be carried out to find a better matching. Iterative computation must be repeated until satisfactory matching is obtained (which is denoted by ∞) between the computed curve ($A_{\text{grav}}^{(\infty)}$ (R), $A_c^{(\infty)}(R)$) and the target curve (A_g , $F(A_g)$) with $A_c = F(A_g)$ by (2).

Thus, the triplet [$A_{\text{grav}}^{(\infty)}(R)$, $A_c^{(\infty)}(R)$, $A_{\text{DM}}^{(\infty)}(R)$], $R \in [R_1, R_2]$, has been found from the computation for the three selected galaxies NGC3198, NGC6503 and NGC3741. Those are plotted in the diagram of Fig. 4 of the main article shown by the blue sequence of dots marked with **c**.

Matching with the curve $A_c = F(A_g)$ marked with **a** is **remarkable**.

The curves **a** and **c** are also given in the diagram of §4.2 given above.

7 *The results show an excellent match of the present predictions with the observed finding by McGaugh, Lelli & Schombert (2016). This matching implies that **the present approach has captured an essential aspect of the galactic dark-matter effect**.*

8 Concerning the physical mechanism, this study implies that **the orbital hyper-speed of order 40 ~ 200 km/sec observed in the halos is maintained by a new force of fluid Lorentz force working in galactic space**. This gives rise to the dark matter effect of unseen objects in the halo of galactic space

9 Supplementary Note: **On the Z-dependence of potentials Φ_g , Ψ and A**

Regarding the Z-dependence of the gravitational potential Φ_g , governed by the Poisson equation $\nabla^2 \Phi_g = 4\pi G \rho$, the Appendix B of the reference [4] describes its mathematical analysis in details. To represent the Z-dependence, the potential is expressed by the form $\Phi_g(Z, R) = f(Z) Q_g(R)$ and the density $\rho(Z, R) = f(Z) \rho_D(R)$, with $f(Z) = \exp[-Z^4/\Delta^4]$, using a constant Δ . Namely, the gravity potential Φ_g and the density ρ are characterized with an exponential behavior of the factor $\exp[-Z^4/\Delta^4]$. This factorization of the function $\Phi_g(Z, R)$ led to the equation (II) of §5, where the term $\partial_Z^2 \Phi_g$ vanishes over the disk-plane $Z=0$.



Next, let us consider the equation (III) of §5: $\partial_R (R^{-1} \partial_R \Psi) = -\mu_* \rho_0(R) V(R)$, which was derived from the equation $\nabla \times \mathbf{b} = \mu_* \rho \mathbf{v}$ where $\Psi = RA$. The variable $A(R)$ is the azimuthal component of the gauge field 3-vector $\mathbf{a} = (0, 0, A)$. The equation $\nabla \times \mathbf{b} = \mu_* \rho \mathbf{v}$ is converted to the form $\nabla^2 \mathbf{a} = -\mu_* \rho \mathbf{v}$ with $\mathbf{v} = (0, 0, V)$. Its azimuthal component is given by

$$\partial_R (R^{-1} \partial_R \Psi) = -\mu_* j, \quad \text{where } \rho \mathbf{v} = (0, 0, j), \quad j = \rho V.$$

Analogously to the density ρ of the gravity case, let us define the current by $j = f(Z) \rho_0(R) V(R)$. Analogously to the gravity potential Φ_g , we define $A(Z, R) = f(Z) A_D(R)$ and the gauge potential $\psi(Z, R) = f(Z) \Psi(R)$.

The factorization of the functions $A(Z, R)$ and $\psi(Z, R)$ led to the equation (III) of §5, given above, where the term $\partial_Z^2 A$ in $\nabla^2 A$ vanishes over the disk-plane $Z=0$.

END