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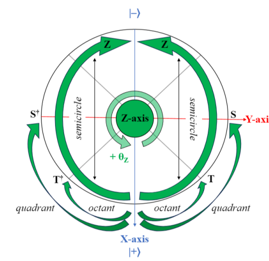
BSA: The Bloch Sphere Approach as a Geometrical Design Tool for Building Cost-Effective Quantum Gates V.2



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Ali Al-Bayaty[1], Marek Perkowski[1]
[1]Portland State University
Ali Al-Bayaty: Department of Electrical and Computer Engineering
Marek Perkowski: Department of Electrical and Computer Engineering


Ali Al-Bayaty
Portland State University

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We use this protocol and it's working

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Disclaimer

Abstract

The Bloch sphere is a geometrical 3D sphere that visualizes the states of a qubit after a series of quantum gates are applied to it. In quantum computing, the Bloch sphere is mainly used as a **geometrical visualization (and verification) tool**.

On the other hand, in this protocol, we introduce the Bloch sphere as a **geometrical design tool** for building cost-effective quantum gates based on their rotational quantum operations in the XY, XZ, and/or YZ planes, which are the 2D circular top and side views of the Bloch sphere. Collectively, the Bloch sphere and its planes are termed the **Bloch sphere approach (BSA)**.

With the BSA, various generic and cost-effective quantum gates and libraries are designed for IBM quantum computers, using the symmetrical and semi-symmetrical structures [1-5], Clifford+T gates, and IBM native "basis" gates ($\sqrt{X}$, X, RZ, and $CNOT$). Our designed generic and cost-effective quantum gates and libraries are listed as follows, where 2 $\leq$ *n* $\leq$ 5 qubits.

1. Quantum libraries: GALA-*n* [3, 6] and CALA-*n* [4, 7], which have become part of the IBM Qiskit ecosystem [8]
2. *n*-bit Toffoli gate [2-4]
3. *n*-bit Boolean gates (AND, NAND, OR, NOR, implication, and inhibition) [3, 4]
4. *n*-bit controlled-$\sqrt{X}$ (CV) and controlled-$\sqrt{X}^{\dagger}$ ($CV^{\dagger}$) gates [3, 4]
5. *n*-bit Fredkin gate [3, 4]
6. *n*-bit Miller gate [3, 4]
7. Boolean-Phase SWAP gate: *p*-SWAP [4, 5]

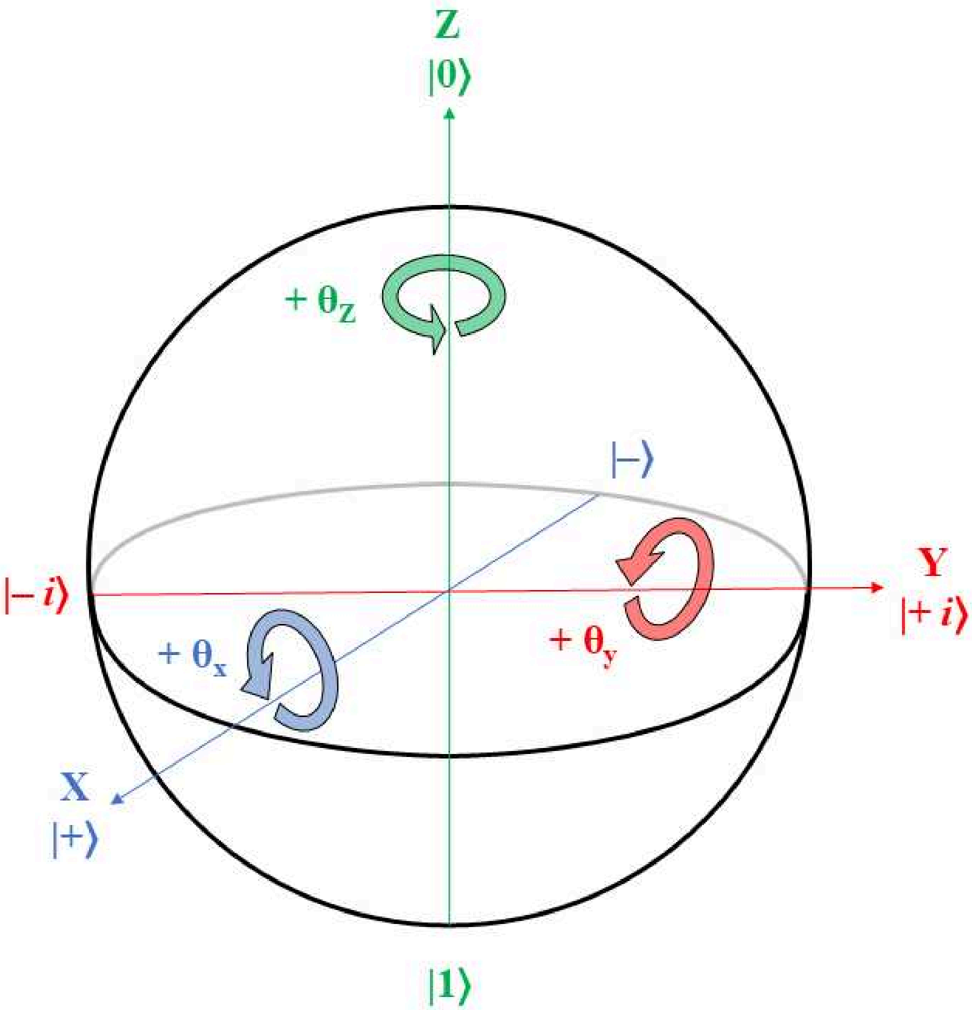
Because the quantum operations of all IBM native gates mainly rotate around the X-axis and Z-axis of the Bloch sphere, the XY-plane of the Bloch sphere is utilized here for the BSA. However, the BSA can also be utilized to build generic and cost-effective quantum gates for other quantum computers, e.g., Intel, Google, and Rigetti, using other projectional planes of the Bloch sphere, i.e., the XZ and YZ planes, based on the supported native gates for such quantum computers. Therefore, in this protocol, we introduce the BSA as a generic and open geometrical framework for prospective quantum computing research for building interesting and innovative quantum gates and circuits.

Troubleshooting

Preliminary Notes

1

Note



The Bloch sphere consists of three axes (X-axis in blue, Y-axis in red, and Z-axis in green) with their corresponding rotational angles (θ_x, θ_y, and θ_z), where "+θ" denotes a counterclockwise rotational angle and "−θ" denotes a clockwise rotational angle, in radians.

2

Note

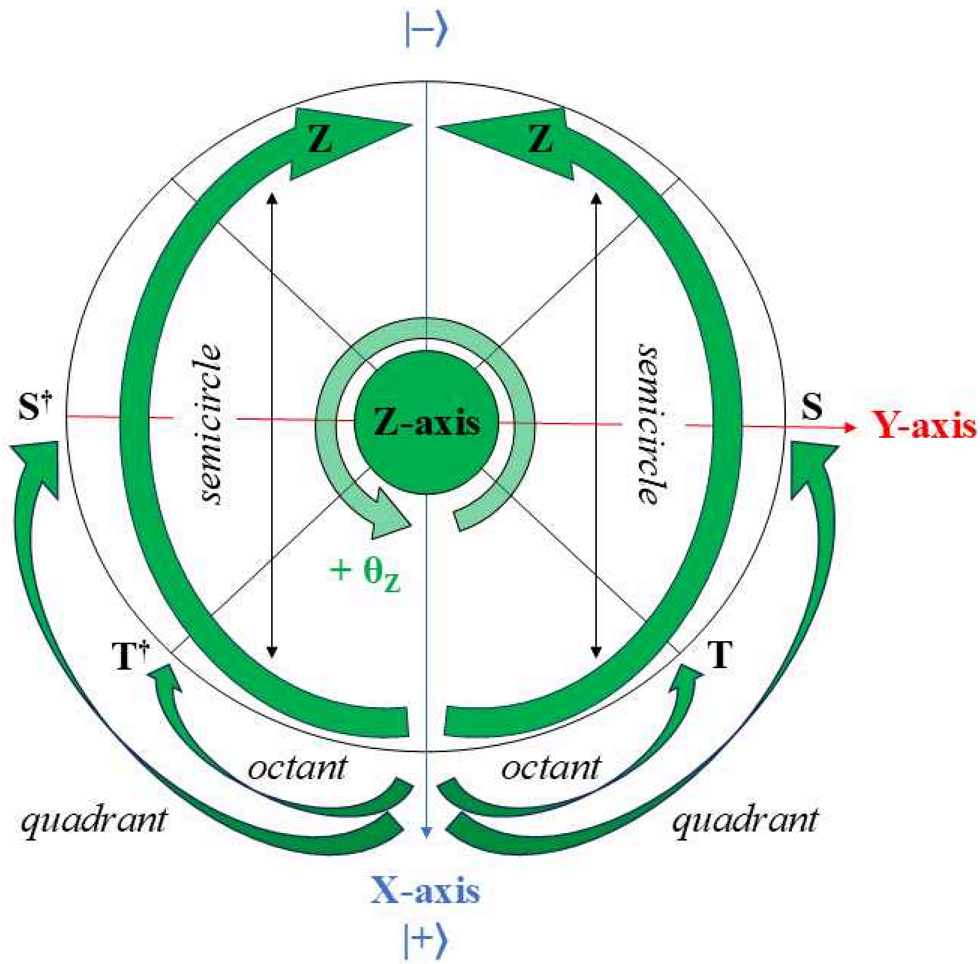
1. The Clifford+T gates are $\{I,\ X,\ Y,\ Z,\ H, \sqrt{X}, \sqrt{X}^{\dagger},\ S,\ S^{\dagger},\ T,\ T^{\dagger},\ CNOT\ (CX),\ CY,\ CZ,\ SWAP\}$.
2. For the BSA, based on IBM native gates, we limit the Clifford+T gates to $\{X,\ Z,\ H, \sqrt{X},\ S,\ S^{\dagger},\ T,\ T^{\dagger},\ CNOT\}$, which is denoted by $\mathbf{CTG_0}$.
3. The gates of $\mathbf{CTG_0}$ rotating around the X-axis of the Bloch sphere are $\{X, \sqrt{X},\ CNOT\}$.
4. The gates of $\mathbf{CTG_0}$ rotating around the Z-axis of the Bloch sphere are $\{Z,\ S,\ S^{\dagger},\ T,\ T^{\dagger}\}$.
5. The H gate of $\mathbf{CTG_0}$ is primarily used in the BSA, to transform the state of a qubit from the Z-axis ($|0\rangle$ or $|1\rangle$) to the XY-plane ($|+\rangle$ or $|-\rangle$), respectively, and vice versa.
6. In general, the H, $\sqrt{X}$, and $\sqrt{X}^{\dagger}$ can be used as superposition gates. However, in the BSA, different geometrical analyses should be taken into account when using $\sqrt{X}$ and $\sqrt{X}^{\dagger}$ as superposition gates to build cost-effective gates for various quantum computers.
7. For $n \geq 2$ qubits, an n-bit quantum gate has n−1 controls (input qubits) and one target (output qubit), except for the SWAP and Fredkin gates.
8. In the BSA, all controls are connected to the target using $CNOT$ gates, and the target has a defined number of $\mathbf{CTG_0}$ gates $\{H,\ X, \sqrt{X},\ Z,\ S,\ S^{\dagger},\ T,\ T^{\dagger}\}$. Every $CNOT$ gate flips the target's state in the perimeter of the XY-plane by $+\ \pi$ radians, i.e., around the X-axis of the Bloch sphere, when its control is in the $|1\rangle$ state.

3

Note

The XY-plane of the Bloch sphere is divided into segments to represent the rotational quantum operations of Clifford+T gates and IBM native gates around the Z-axis as follows.

1. The **semicircle** segment is half of the XY-plane, to represent the quantum rotations of Z gates. Such that, the XY-plane has two semicircles.
2. The **quadrant** segment is one-fourth of the XY-plane, to represent the quantum rotations of S and $S^\dagger$ gates. Such that, the XY-plane has four quadrants.
3. The **octant** segment is one-eighth of the XY-plane, to represent the quantum rotations of T and $T^\dagger$ gates. Such that, the XY-plane has eight octants.

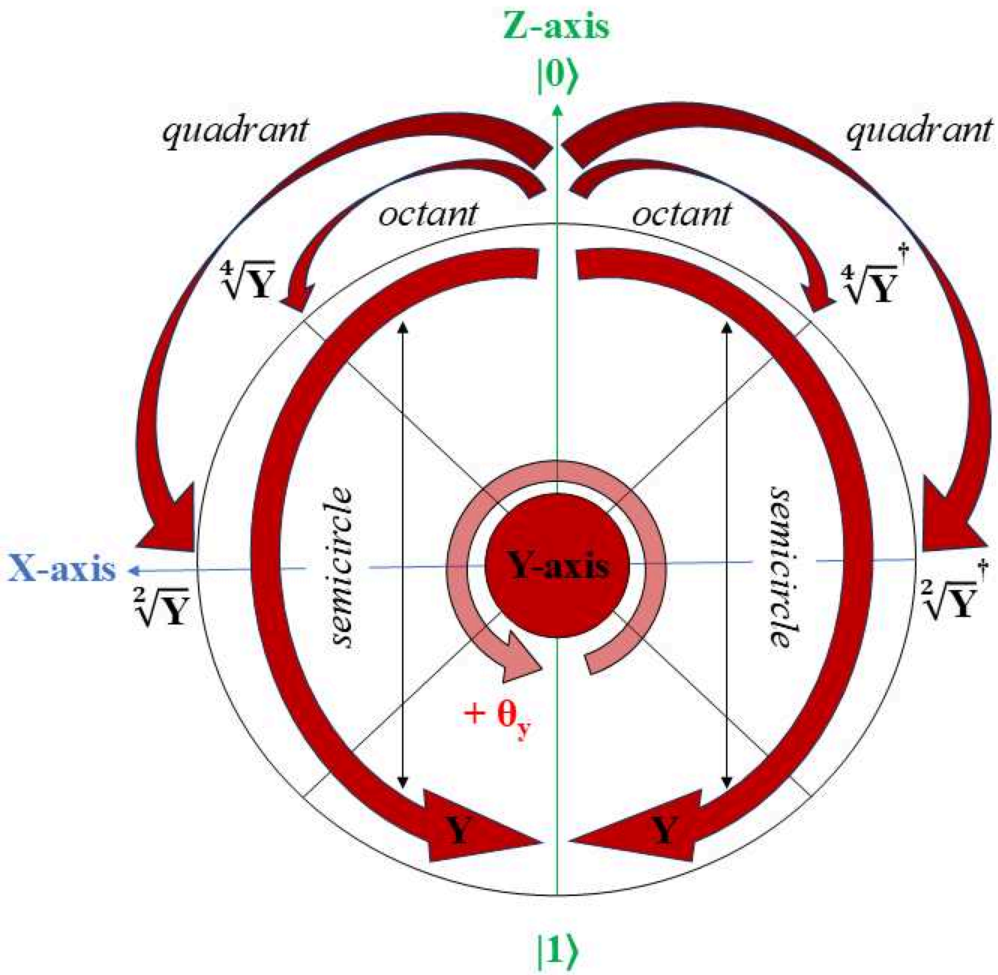


The XY-plane visualizes the rotational quantum operations of Cifford+T gates $\{Z,\ S,\ S^\dagger,\ T,\ T^\dagger\}$ around the Z-axis of the Bloch sphere, when a qubit is initially set to the $|+\rangle$ state.

4

Note

Similarly, the XZ-plane of the Bloch sphere is divided into segments (semicircles, quadrants, and octants) to represent the rotational quantum operations based on the native gates of a quantum computer, as illustrated below.

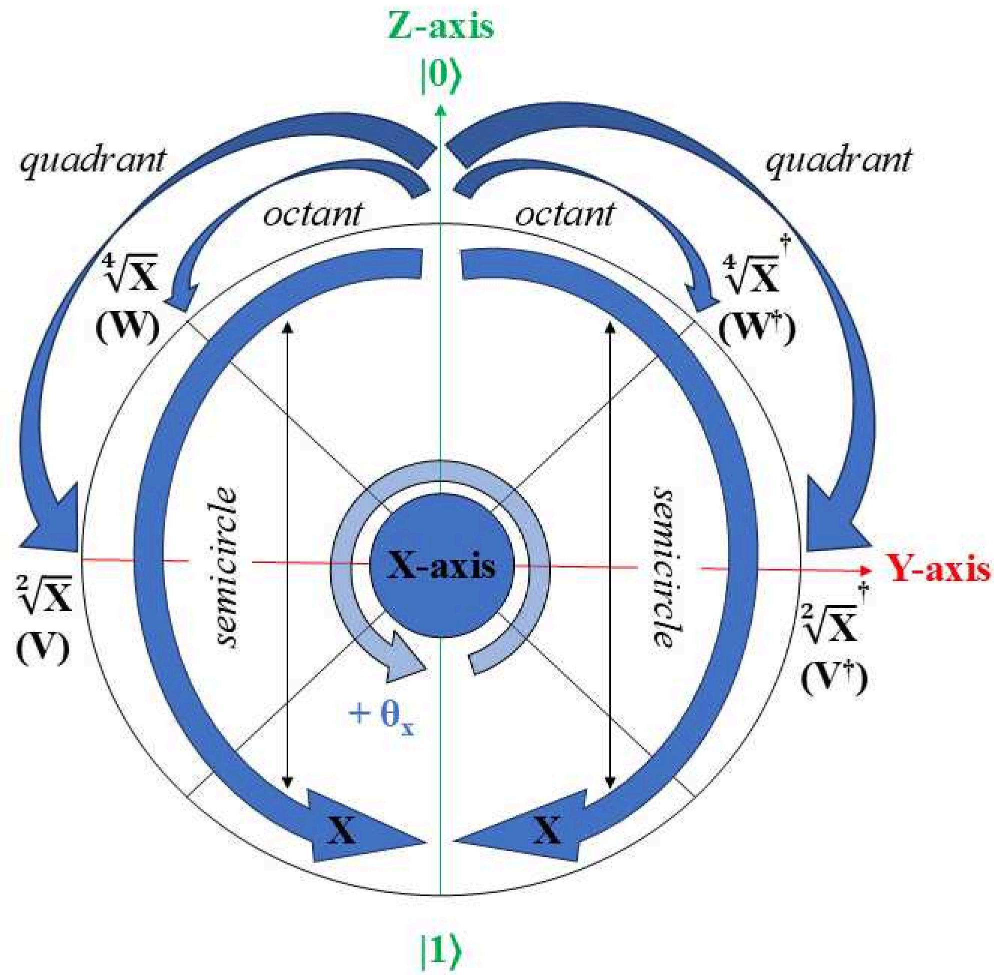


The XZ-plane visualizes the rotational quantum operations of native gates for a quantum computer around the Y-axis of the Bloch sphere, when a qubit is initially set to the $|0\rangle$ state.

5

Note

Similarly, the YZ-plane of the Bloch sphere is divided into segments (semicircles, quadrants, and octants) to represent the rotational quantum operations based on the native gates of a quantum computer, as demonstrated below.

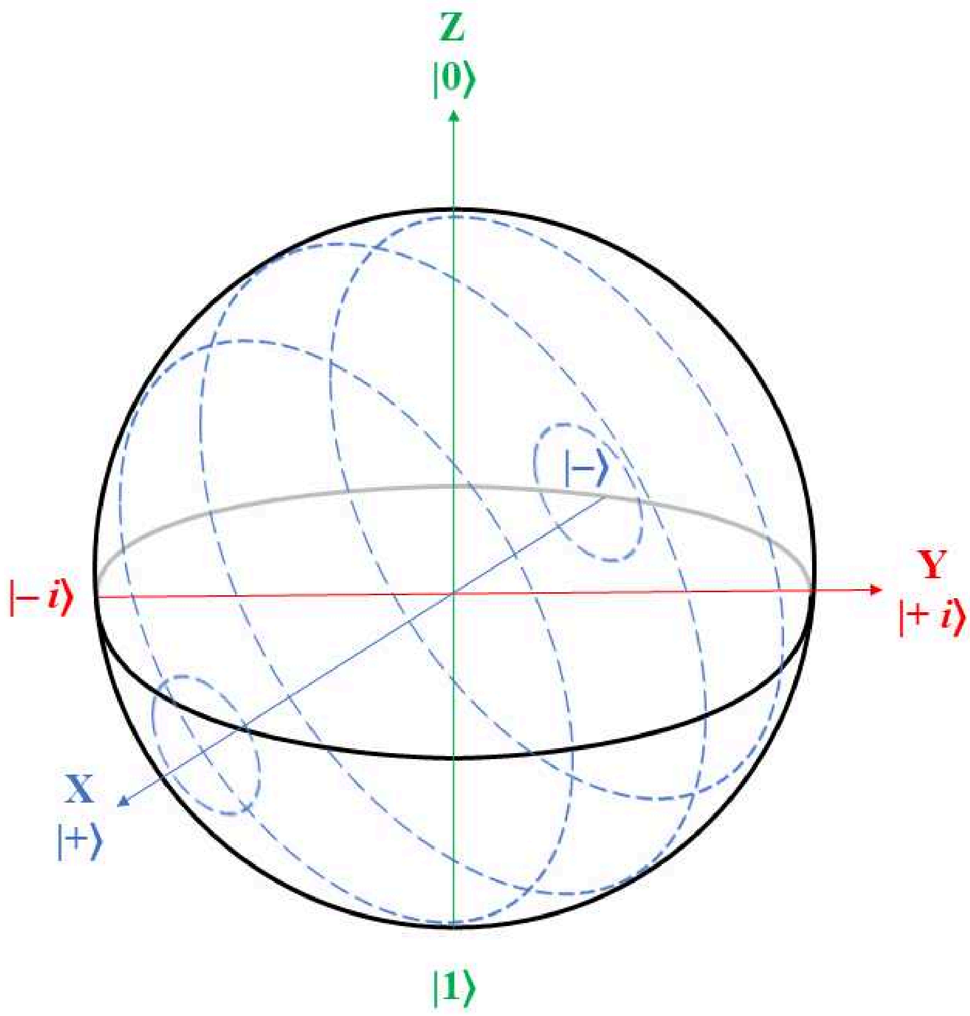


The YZ-plane visualizes the rotational quantum operations of native gates for a quantum computer around the X-axis of the Bloch sphere, when a qubit is initially set to the $|0\rangle$ state.

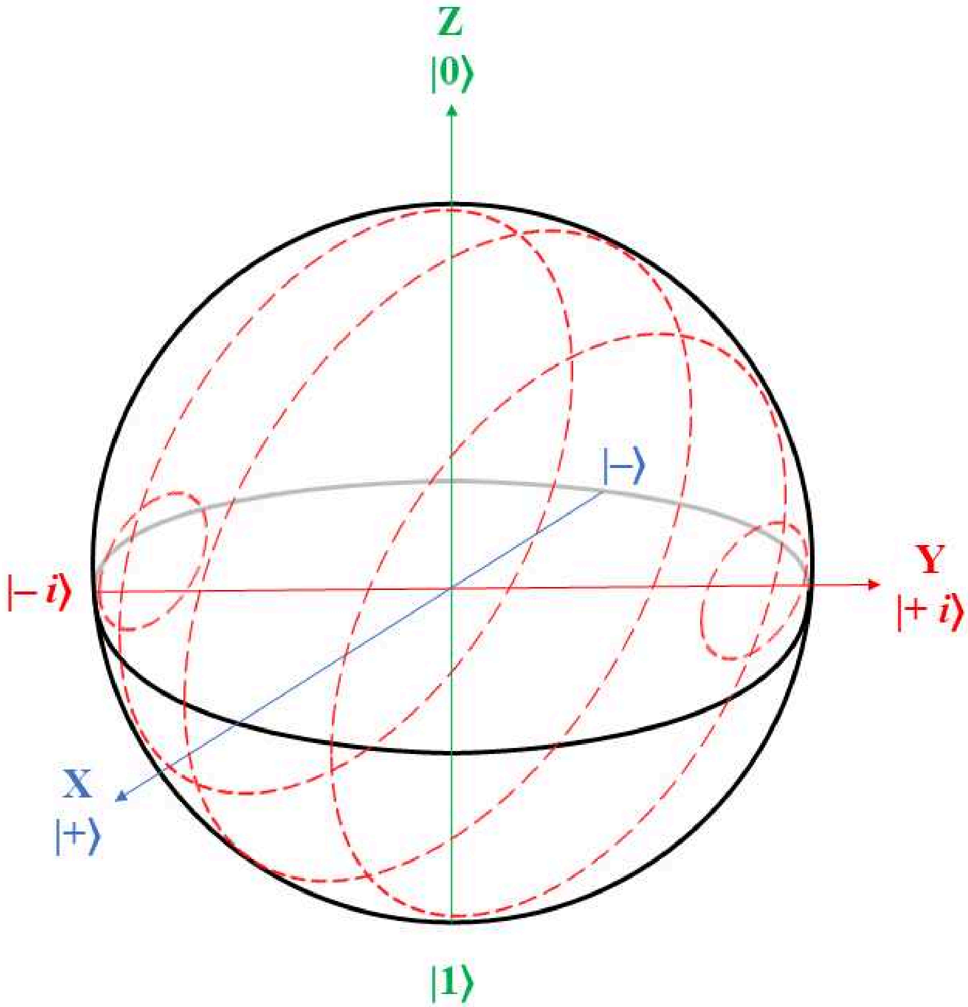
6

Note

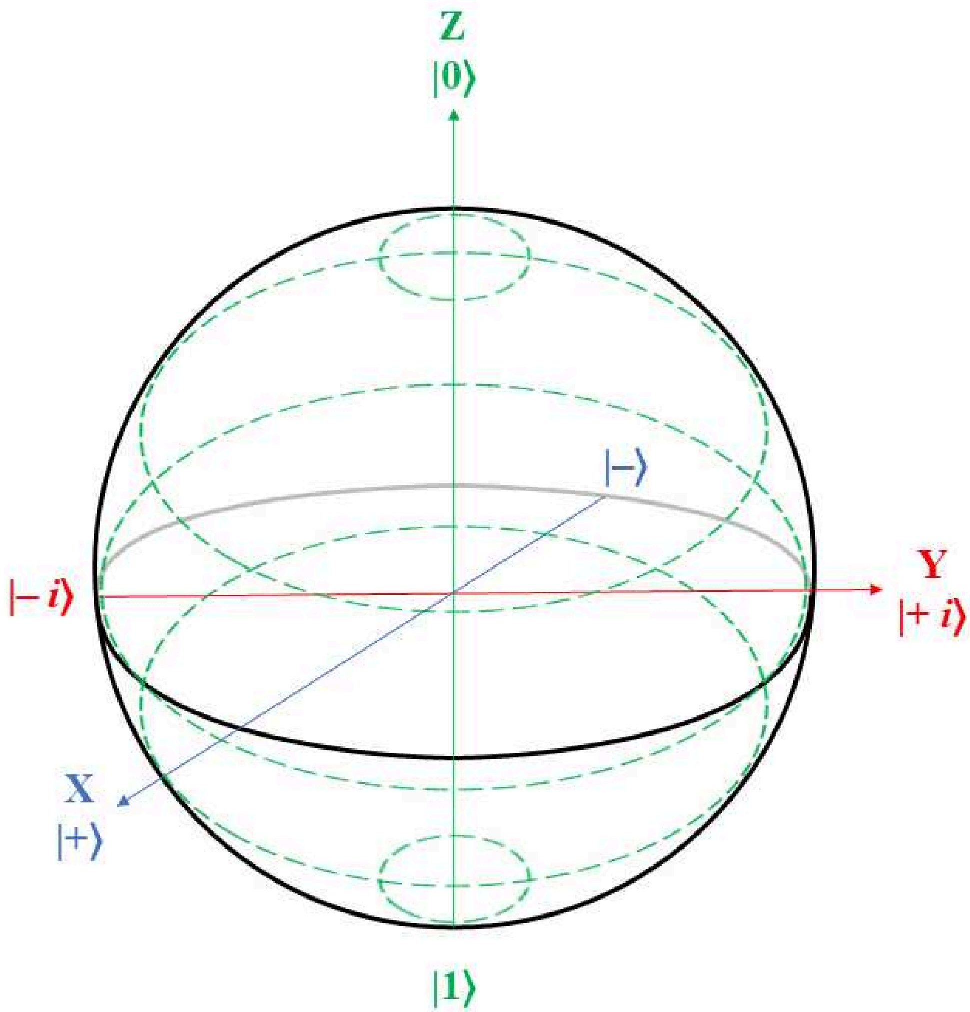
In general, the Bloch sphere approach (BSA) is a generic and open geometrical framework for building cost-effective quantum gates, based on the Clifford+T and native gates of any quantum computer. Hence, various segments (semicircles, quadrants, octants, etc.) and different projectional planes of the Bloch sphere can be combined together, as depicted below based on the utilized axes of the Bloch sphere.



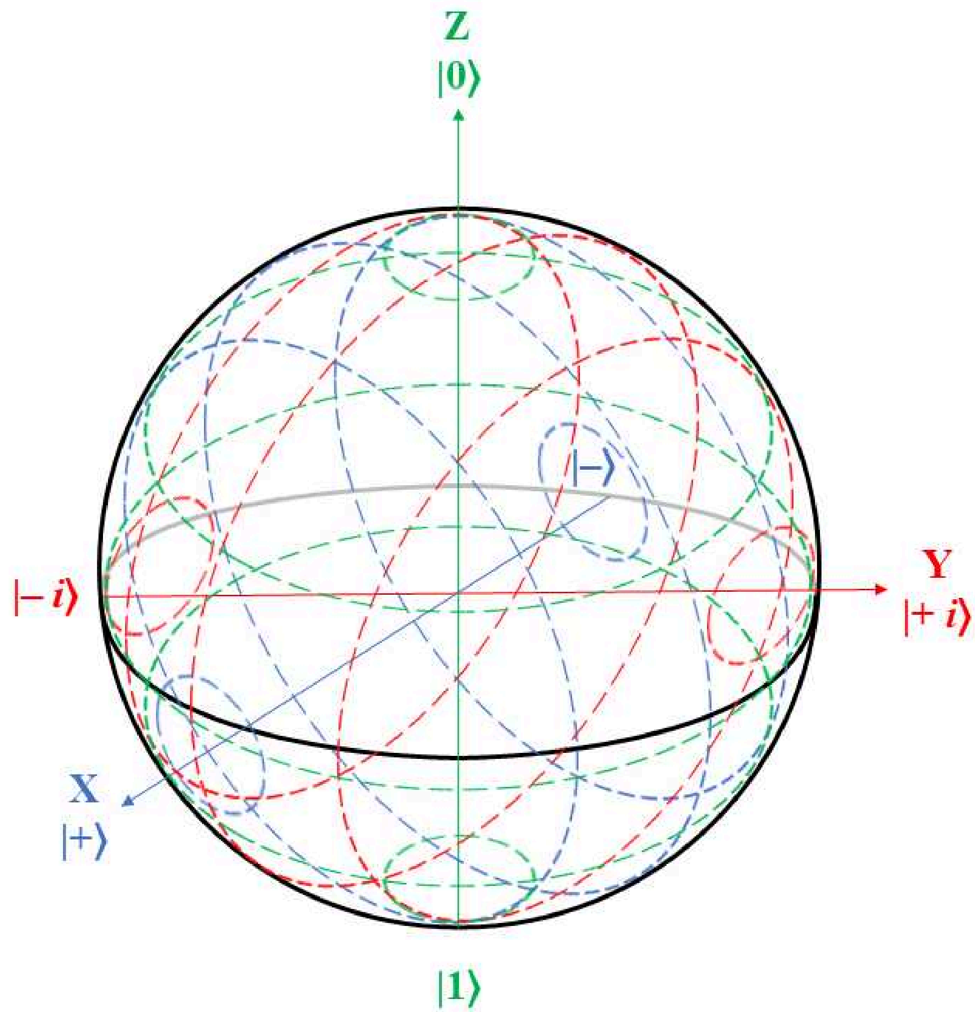
Various segments (semicircles, quadrants, octants, etc.) and projectional planes around the X-axis of the Bloch sphere.



Various segments (semicircles, quadrants, octants, etc.) and projectional planes around the Y-axis of the Bloch sphere.



Various segments (semicircles, quadrants, octants, etc.) and projectional planes around the Z-axis of the Bloch sphere.



Various segments (semicircles, quadrants, octants, etc.) and projectional planes around all three axes of the Bloch sphere.

The BSA Protocol (for IBM quantum computers)

7 For symmetrical and semi-symmetrical structures [1-5], transform the target's state of an *n*-bit quantum gate from the Z-axis of the Bloch sphere into the XY-plane using one H gate, where $n \geq 2$ qubits. Note that the target is initially set to either $|0\rangle$ or $|1\rangle$ state.

8 For the target, define all Clifford+T gates (based on IBM native gates) as the set: $\mathbf{CTG_0}$ = $\{H, \sqrt{X},\ X,\ Z,\ S,\ S^{\dagger},\ T,\ T^{\dagger},\ CNOT\}.$

9 For the target, define all segments of the XY-plane as the set: $\mathbf{SEG_0}$ = {*semicircles*, *quadrants*, *octants*}.

10 If the target is not controlling other qubits, then **CTG$_0$** is limited to the set: **CTG$_1$** = $\{H, \sqrt{X},\ X,\ Z,\ S,\ S^{\dagger},\ T,\ T^{\dagger}\}$. Otherwise, **CTG$_1$** = **CTG$_0$**.

11 Since **SEG$_0$** only identifies quantum gates rotating around the Z-axis of the Bloch sphere, then **CTG$_1$** is limited to the set: **CTG$_2$** = $\{Z,\ S,\ S^{\dagger},\ T,\ T^{\dagger}\}$.

> Note
>
> Here, we assume that the 1-bit quantum rotations around the X-axis of the Bloch sphere are not required for the target. Otherwise, this step is negligible and **CTG$_2$** = **CTG$_1$**, but this will affect the selection of quantum rotations in the next steps!

12 Let $nCNOT$ counts the total number of $CNOT$ gates from the controls to the target, and the new sets of **CTG** and **SEG** will be defined as follows.

12.1 If $nCNOT\ =\ 1$, then **CTG$_3$** = $\{S,\ S^{\dagger},\ T,\ T^{\dagger}\}$, and **SEG$_1$** = {*quadrants*, *octants*}.

12.2 If $nCNOT\ =\ 2$, then a set of IBM native RZ gates defines **CTG$_3$** = {$RZ_1(\theta)$, $RZ_2(\theta)$, ...}, where θ $\leq \pm\ \dfrac{\pi}{3}$ for arbitrary **SEG$_1$**.

12.3 If $nCNOT\ =\ 3$, then **CTG$_3$** = $\{T,\ T^{\dagger}\}$, and **SEG$_1$** = {*octants*}.

12.4 If $nCNOT\ >\ 3$, then a set of IBM native RZ gates defines **CTG$_3$** = {$RZ_1(\theta)$, $RZ_2(\theta)$, ...}, where θ $\leq \pm\ \dfrac{\pi}{nCNOT\ +\ 1}$ for arbitrary **SEG$_1$**.

13 Based on Step 12, the $nCNOT$ and permutative gates of **CTG$_3$** define the desirable quantum operation (logic and behavior) of an *n*-bit quantum gate, which can be designed and visualized based on the **SEG$_1$** of the XY-plane.

14 Re-transform the final target's state of an *n*-bit quantum gate from the XY-plane into the Z-axis of the Bloch sphere using another H gate.

15 Finally, the re-transformed target's state is considered the output of such a quantum gate. For instance, the target in the $|1\rangle$ state indicates a solution (*True*), while its $|0\rangle$ state indicates a non-solution (*False*).

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