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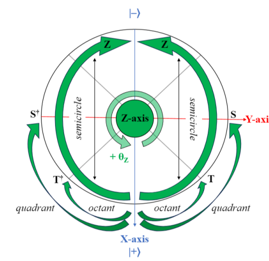
BSA: The Bloch Sphere Approach as a Geometrical Design Tool for Building Cost-Effective Quantum Gates V.1



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We use this protocol and it's working

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Abstract

The Bloch sphere is a geometrical 3D sphere that visualizes the states of a qubit after a series of quantum gates are applied to it. For that, the Bloch sphere is mainly used as a **geometrical visualization (and verification) tool**.

On the other hand, in this protocol, we introduce the Bloch sphere as a **geometrical design tool** for building cost-effective quantum gates based on their rotational quantum operations in the XY-plane, which is the 2D circular top-view of the Bloch sphere. Collectively, the Bloch sphere and its XY-plane are termed the **Bloch sphere approach (BSA)**.

With the BSA, various generic and cost-effective quantum gates and libraries are designed for IBM quantum computers, using the symmetrical and semi-symmetrical structures [1-5], Clifford+T gates, and IBM native gates ($\sqrt{X}$, X, RZ, and $CNOT$), as follows.

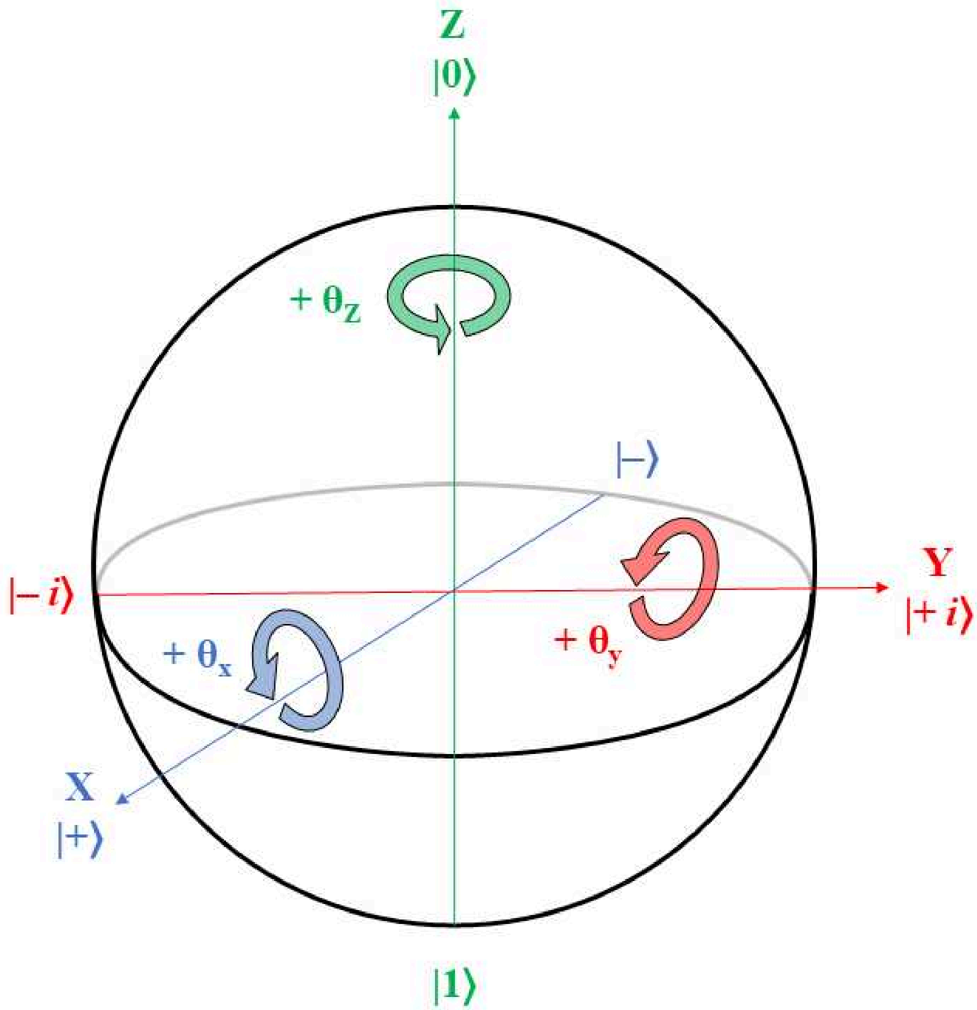
1. Quantum libraries (GALA-*n* [3, 6] and CALA-*n* [4, 7]), which have become part of the IBM Qiskit ecosystem [8]
2. *n*-bit Toffoli gate [2-4]
3. *n*-bit Boolean gates (AND, NAND, OR, NOR, implication, and inhibition) [3, 4]
4. *n*-bit controlled-$\sqrt{X}$ (CV) and controlled-$\sqrt{X}^\dagger$ ($CV^\dagger$) gates [3, 4]
5. *n*-bit Fredkin gate [3, 4]
6. *n*-bit Miller gate [3, 4]
7. Boolean-Phase SWAP gate (*p*-SWAP) [4, 5]

Because the quantum operations of all IBM native "basis" gates mainly rotate around the X-axis and Z-axis of the Bloch sphere, we propose the XY-plane of the Bloch sphere for the BSA. However, the BSA can also be utilized to build generic and cost-effective quantum gates for other quantum computers, e.g., Intel, Google, and Rigetti, using different projectional planes of the Bloch sphere, e.g., the XZ-plane and the YZ-plane, based on the supported native gates for such quantum computers. Therefore, in this protocol, we introduce the BSA as an open geometrical framework for prospective quantum computing research.

Preliminary Notes

1

Note



The Bloch sphere consists of three axes (X-axis in blue, Y-axis in red, and Z-axis in green) with their corresponding rotational angles (θ_x, θ_y, and θ_z), where "+θ" denotes a counterclockwise rotational angle and "−θ" denotes a clockwise rotational angle, in radians.

2

Note

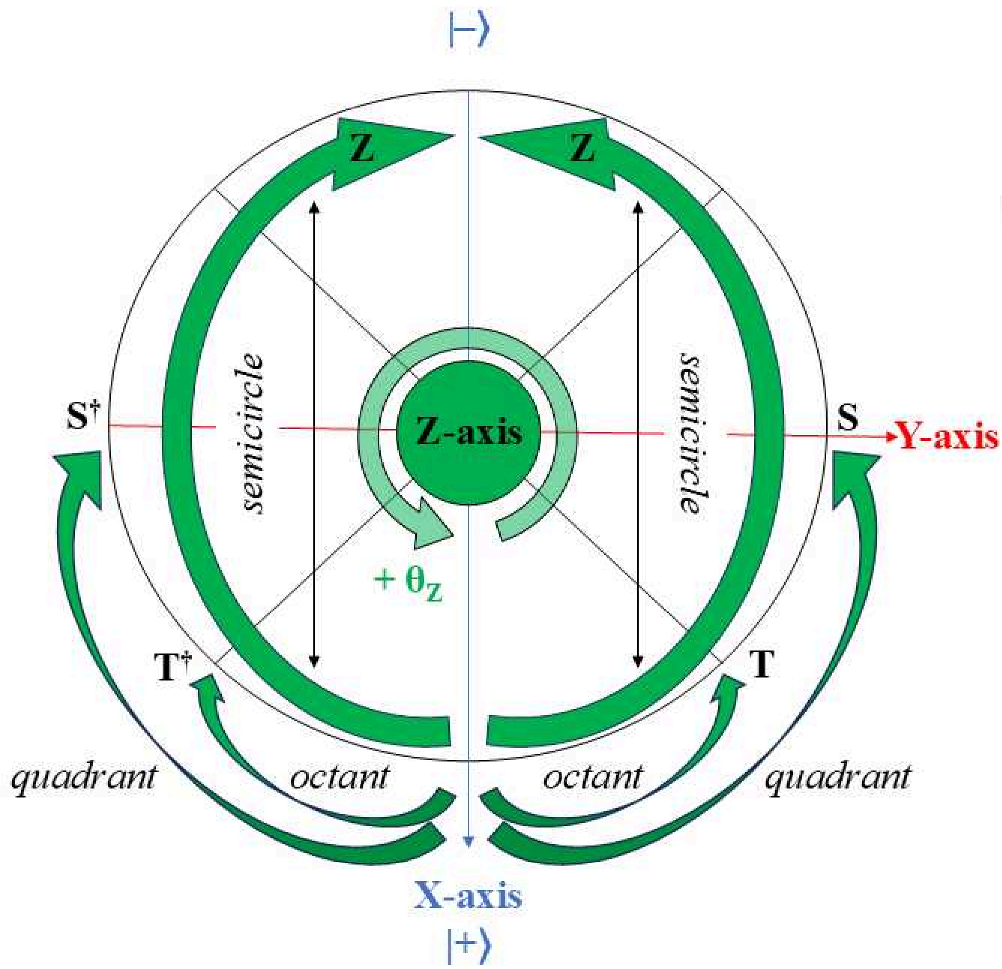
1. The Clifford+T gates are $\{I,\ X,\ Y,\ Z,\ H, \sqrt{X}, \sqrt{X}^{\dagger},\ S,\ S^{\dagger},\ T,\ T^{\dagger},\ CNOT\ (CX),\ CY,\ CZ,\ SWAP\}$.
2. For the BSA, based on IBM native gates, we limit the Clifford+T gates to $\{X,\ Z,\ H, \sqrt{X},\ S,\ S^{\dagger},\ T,\ T^{\dagger},\ CNOT\}$, which is denoted by **CTG$_0$**.
3. The gates of **CTG$_0$** rotating around the X-axis of the Bloch sphere are $\{X, \sqrt{X},\ CNOT\}$.
4. The gates of **CTG$_0$** rotating around the Z-axis of the Bloch sphere are $\{Z,\ S,\ S^{\dagger},\ T,\ T^{\dagger}\}$.
5. The H gate of **CTG$_0$** is primarily used in the BSA, to transform the state of a qubit from the Z-axis ($|0\rangle$ or $|1\rangle$) to the XY-plane ($|+\rangle$ or $|-\rangle$), respectively, and vice versa.
6. In general, the H, $\sqrt{X}$, and $\sqrt{X}^{\dagger}$ can be used as superposition gates. However, in the BSA, different geometrical analyses should be taken into account when using $\sqrt{X}$ and $\sqrt{X}^{\dagger}$ as superposition gates to build cost-effective gates for various quantum computers.
7. For $n \geq 2$ qubits, an n-bit quantum gate has $n-1$ controls (input qubits) and one target (output qubit), except for the SWAP and Fredkin gates.
8. In the BSA, all controls are connected to the target using $CNOT$ gates, and the target has a defined number of **CTG$_0$** gates $\{H,\ X, \sqrt{X},\ Z,\ S,\ S^{\dagger},\ T,\ T^{\dagger}\}$. Every $CNOT$ gate flips the target's state in the perimeter of the XY-plane by $+\ \pi$ radians, i.e., around the X-axis of the Bloch sphere, when its control is only in the $|1\rangle$ state.

3

Note

The XY-plane of the Bloch sphere is divided into segments to represent the rotational quantum operations of Clifford+T gates and IBM native gates around the Z-axis as follows.

1. The **semicircle** segment is half of the XY-plane, to represent the quantum rotations of Z gates. Such that, the XY-plane has two semicircles.
2. The **quadrant** segment is one-fourth of the XY-plane, to represent the quantum rotations of S and $S^\dagger$ gates. Such that, the XY-plane has four quadrants.
3. The **octant** segment is one-eighth of the XY-plane, to represent the quantum rotations of T and $T^\dagger$ gates. Such that, the XY-plane has eight octants.



The XY-plane visualizes the rotational quantum operations of Cifford+T gates $\{Z,\ S,\ S^\dagger,\ T,\ T^\dagger\}$ around the Z-axis of the Bloch sphere, when a qubit is initially set to the $|+\rangle$ state.

The BSA Protocol

4 Transform the target's state of an *n*-bit quantum gate from the Z-axis of the Bloch sphere into the XY-plane using the H gate, where *n* $\geq$ 2 qubits.

5 For the target, define all Clifford+T gates (based on IBM native gates) as a set: $\mathbf{CTG_0}$ = $\{H, \sqrt{X},\ X,\ Z,\ S,\ S^\dagger,\ T,\ T^\dagger,\ CNOT\}$.

6 For the target, define all segments of the XY-plane as a set: $\mathbf{SEG_0}$ = {*semicircles*, *quadrants*, *octants*}.

7 If the target is not controlling other qubits, then $\mathbf{CTG_0}$ is limited to a set: $\mathbf{CTG_1}$ = $\{H, \sqrt{X},\ X,\ Z,\ S,\ S^\dagger,\ T,\ T^\dagger\}$. Otherwise, $\mathbf{CTG_1}$ = $\mathbf{CTG_0}$.

8 Since $\mathbf{SEG_0}$ only identifies Clifford+T gates rotating around the Z-axis of the Bloch sphere, then $\mathbf{CTG_1}$ is limited to a set: $\mathbf{CTG_2}$ = $\{Z,\ S,\ S^\dagger,\ T,\ T^\dagger\}$.

> Note
>
> Here, we assume that the 1-bit quantum rotations around the X-axis of the Bloch sphere are not required for the target. Otherwise, this step is negligible, but this will affect the selection of quantum rotations in the next steps!

9 The number of $CNOT$ gates ($nCNOT$) from the controls to the target defines the new sets of **CTG** and **SEG**, as in the following:

9.1 If $nCNOT\ =\ 1$, then $\mathbf{CTG_3}$ = $\{S,\ S^\dagger,\ T,\ T^\dagger\}$, and $\mathbf{SEG_1}$ = {*quadrants*, *octants*}.

9.2 If $nCNOT\ =\ 2$, then a set of IBM native RZ gates will be used as $\mathbf{CTG_3}$ = RZ(θ), where θ = $\pm\dfrac{\pi}{3}$, with arbitrary $\mathbf{SEG_1}$.

9.3 If $nCNOT\ =\ 3$, then $\mathbf{CTG_3}$ = $\{T,\ T^\dagger\}$, and $\mathbf{SEG_1}$ = {*octants*}.

9.4 If $nCNOT\ >\ 3$, then a set of IBM native RZ gates will be used as $\mathbf{CTG_3}$ = RZ(θ), where θ = $\pm\dfrac{\pi}{nCNOT\ +\ 1}$, with arbitrary $\mathbf{SEG_1}$.

10 The chosen $nCNOT$ and the permutative values of $\mathbf{CTG_3}$ define the desired quantum operation (logic and behavior) of an *n*-bit quantum gate.

11 Re-transform the target's state of an *n*-bit quantum gate from the XY-plane into the Z-axis of the Bloch sphere using the H gate, and the final target's state is considered the output of such a gate, as the $|1\rangle$ state (a solution) or the $|0\rangle$ state (a non-solution).

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